MRTD Modeling of Nonlinear Pulse Propagation

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Abstract- The nonlinear pulse propagation in optical fiber filters is modeled by a multiresolution time domain (MRTD) scheme based on the expansion in scaling functions. As for the case of the linear modeling of microwave structures, the MRTD scheme exhibits advantages over Yee's FDTD scheme, since it allows to reduce the mesh size by a factor of five per dimension. This is demonstrated by modeling the pulse compression with both MRTD and FDTD schemes.

1 Introduction

Recently, new multiresolution time domain (MRTD) schemes based on othornormal wavelet expansions have been applied to the analysis of microwave structures. It has been shown that the MRTD schemes exhibit the capability of approximating the exact solution for sampling rates approaching the Nyquist limit [1]. Thus the minimum discretization for accurate MRTD results is close to two points per wavelength, while it is usually about ten points per wavelength for accurate FDTD results. This explains why the results for FDTD and MRTD presented in [1] exhibit about the same accuracy using a MRTD mesh with five times less grid points per dimension. The objective of this paper is to demonstrate that the same advantages of MRTD over FDTD exist in the case of nonlinear modeling.

While the physics of pulse compression in optical fiber filters is described in detail in [2], this paper concentrates on the computational aspects of solving the nonlinear partial differential equations. In [1], cubic spline Battle-Lemarie scaling and wavelet functions have been used as expansion functions in space domain. Since the use of scaling and wavelet functions as a complete set of basis functions is called multiresolution analysis [3], the resulting time domain schemes have been called multiresolution time domain (MRTD) schemes. Throughout this paper, the electromagnetic fields are represented by an expansion in terms of scaling functions only, thus the resulting scheme is denoted by S-MRTD scheme. In order to obtain a two-step S-MRTD scheme with respect to time, pulse functions are used as expansion and test functions in time domain.

2 The S-MRTD scheme

Since the physics of pulse compression in optical fiber filters is described in detail in [2], we concentrate on the computational aspects of solving the corresponding nonlinear partial differential equations. Assuming a nonlinear polarization and a spatially periodic refractive index as well as making use of the slowly varying envelope approximation, the pulse propagation in nonlinear media may be described by [2]

$$\frac{\partial E_F}{\partial z} + \frac{n_0}{c}\frac{\partial E_F}{\partial t} = \jmath\,\kappa\, E_B\, e^{-2\jmath\Delta\beta\, z} + \jmath\,\gamma\left(|E_F|^2 + 2\,|E_B|^2\right) E_F \quad (1)$$

$$\frac{\partial E_B}{\partial z} - \frac{n_0}{c}\frac{\partial E_B}{\partial t} = -\jmath\,\kappa\, E_F\, e^{2\jmath\Delta\beta\, z} - \jmath\,\gamma\left(|E_B|^2 + 2\,|E_F|^2\right) E_B\,. \quad (2)$$

The terms on the right-hand side cubic in fields describe self-phase modulation, while the linear terms on the left-hand describe the dispersive coupling between the slowly varying electric field components, the forward field E_F and the backward field E_B. The refractive index n_0 represents the refractive index of the nonlinear medium without spatially periodic variation. The coupling constant κ is directly proportional to the amplitude of the cosines variation of the refractive index, while γ is directly proportional to the nonlinear refractive index coefficient. The detuning parameter $\Delta\beta$ is defined as the difference between the propagation constant of a guided mode and the wave number of the grating [2].

Expanding the forward and backward field in terms of scaling functions in space domain and in terms of pulse functions in time domain and proceeding as described in [1], we obtain the partial difference equations

$$\begin{aligned} E^F_{k+1,m} - E^F_{k-1,m} + s\,\boldsymbol{D}_z(E^F_{k,m}) &= 2\,\jmath\,\kappa\, s\,\Delta z \sum_{m'} I^-_{m,m'}\, E^B_{k,m'} \\ &+ 2\,\jmath\,\gamma\, s\,\Delta z \sum_{m'} I_{m,m'}\left(|E^F_{k,m'}|^2 + 2\,|E^B_{k,m'}|^2\right) E^F_{k,m'} \end{aligned}$$

$$E^B_{k+1,m} - E^B_{k-1,m} - s\,\boldsymbol{D}_z(E^B_{k,m}) = 2\,\jmath\,\kappa\, s\,\Delta z \sum_{m'} I^+_{m,m'}\, E^F_{k,m'}$$

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$$+\, 2\jmath\gamma\, s\,\Delta z \sum_{m'} I_{m,m'} \left(|E^B_{k,m'}|^2 + 2\,|E^F_{k,m'}|^2\right) E^F_{k,m'}, \tag{3}$$

where the operator $\boldsymbol{D}_z(E^y_{k,m})$ is given by

$$\boldsymbol{D}_z(E^y_{k,m}) = \sum_{\imath=-15}^{+15} a(i)\; E^y_{k,m+\imath} \quad . \tag{4}$$

The coefficients $E^y_{k,m}$ with $y = F, B$ are the coefficients of the expansions in terms of pulse and scaling functions and s represents the stability factor given by $s = c\Delta t/(n_0 \Delta l)$. The indices m and k are the discrete space and time indices related to the space and time coordinates via $z = m\Delta z$ and $t = k\Delta t$, where Δz and Δt represent the space discretization interval in z–direction and the time discretization interval, respectively.

The coefficients $a(i)$ are defined by the integral expression

$$\int_{-\infty}^{+\infty} \phi_m(z)\, \frac{\partial \phi_{m'}(z)}{\partial z}\, dz = \sum_{\imath=-\infty}^{+\infty} a(i)\; \delta_{m+\imath,m'} \tag{5}$$

and may be evaluated analytically using the representation of the scaling function in terms of cubic spline functions [4]. The Battle–Lemarie scaling functions incorporated in the MRTD scheme do not have compact but only exponential decaying support and thus, even for large values of i, the coefficients $a(i)$ are not zero. However, the coefficients for larger i are negligible and do not affect the accuracy of the nonlinear MRTD modeling. For this specific example, we neglected the coefficients $a(i)$ for $i > 15$.

The integrals

$$I_{m,m'} = \int_0^L \phi_m(z)\; \phi_{m'}(z)\; dz \tag{6}$$

and

$$I^{\pm}_{m,m'} = \int_0^L \phi_m(z)\; \phi_{m'}(z)\; e^{\pm 2\jmath \Delta\beta\, z} dz \tag{7}$$

take into account that the application of non–localized basis functions like the Battle–Lemarie scaling functions requires a non–localized modeling of material discontinuities. The representation of material discontinuities in terms of scaling functions gives rise to a linear matrix equation as explained in [1] where this technique was used in the modeling of anisotropic dielectric media. More specifically, the integrals $I_{m,m'}$ and $I^{\pm}_{m,m'}$ take in account that the transition from the free space to the nonlinear medium is represented by a step function and by a localized material discontinuity, respectively. The integrals may again be evaluated numerically using the representation of the scaling function in terms of cubic spline functions [4]. Note that due to the symmetry properties of the scaling function the following is true,

$$I_{L/\Delta z-m,L/\Delta z-m'} = I_{m,m'} \tag{8}$$

as well as

$$I^{\pm}_{L/\Delta z-m,L/\Delta z-m'} = I^{\mp}_{m,m'}\; e^{\pm 2\jmath \Delta\beta\, L} \tag{9}$$

and

$$I^{\mp}_{m,m'} = \left(I^{\pm}_{m,m'}\right)^* \quad , \tag{10}$$

where $(I^{\pm}_{m,m'})^*$ represents the conjugate complex of $I^{\pm}_{m,m'}$. In view of the above equations, only one matrix representing $I_{m,m'}$ and one representing $I^{\pm}_{m,m'}$ have to be calculated. For the MRTD simulations in this paper, the integrals $I_{m,m'}$ and $I^{\pm}_{m,m'}$ involving scaling functions located close to the beginning and the end of the nonlinear material were appoximated by 9×9 matrices assuming a discretization, where the maxima of the scaling functions at the beginning and the end of the nonlinear material are placed exactly at $z = 0$ and $z = L$. Right in the nonlinear material, for $0 \ll m, m' \ll L/\Delta z$, the off–diagonal elements can be neglected and we may approximate

$$I_{m,m'} \approx \delta_{m,m'}\, \Delta z \tag{11}$$

and

$$I^{\pm}_{m,m'} \approx \delta_{m,m'}\, \Delta z\, e^{\pm 2\jmath \Delta\beta\, m\,\Delta z} \quad . \tag{12}$$

The boundary conditions at the beginning of the nonlinear material at $z = 0$ and at the end of the nonlinear material at $z = L$ are given by

$$|E_F(0,t)|^2 = A\, e^{-t^2/\alpha^2} \tag{13}$$

and

$$E_B(L,t) = 0 \tag{14}$$

as well as

$$\left.\frac{\partial E_B}{\partial z}\right|_{z=0} = \frac{n_0}{c}\left.\frac{\partial E_B}{\partial t}\right|_{z=0} \tag{15}$$

and

$$\left.\frac{\partial E_F}{\partial z}\right|_{z=L} = -\frac{n_0}{c}\left.\frac{\partial E_F}{\partial t}\right|_{z=L} \quad . \tag{16}$$

Since the MRTD scheme is based on the expansions of the fields in terms of non–localized basis functions, the implementation of the boundary conditions (14) is not as straightforward as for FDTD. The boundary condition (14) requires the backward field to be zero, which is the same as considering a perfect electric conductor (PEC) at $z = L$ for the backward field only. Since the two partial differential equations (1) and (2) decouple for linear materials, we may satisfy eq. (14) by adding a slice of free space at the end of the nonlinear material for the backward field only and terminating the slice with a PEC. Then the image principle [1] is applied to model the PEC at $z = M\Delta z$. This is equivalent to terminating the mesh at $z = M\Delta z$ and making use of the symmetry relation

$$E^F_{k,M+m} = -E^F_{k,M-m} \tag{17}$$

for the expansions coefficients outside the mesh.

The radiating boundary conditions (15) and (16) have been satisfied by applying the perfectly matched layer (PML) technique [5]. To implement the PML technique presented for FDTD, we assume that the conductivity is

$L/\Delta z$	FDTD	MRTD
100		10.407
200	7.969	10.623
500	10.189	
1000	10.562	

Table 1: The peak values of the intensity of the transmitted pulse.

given in terms of scaling functions instead of pulse functions with respect to space. Then the partial difference equations for modeling the absorbing material used in [5] may be directly applied for MRTD. For the spatial distribution of the conductivity in the absorbing layer, we assume that the amplitudes of the scaling functions have a parabolic distribution. The mesh at the end of the absorbing layer is terminated by a PEC which is again modeled by the image principle.

Fig. 1 shows the intensity $|E_F(z,t)|^2$ of the transmitted pulse modeled by the MRTD scheme using a Gaussian excitation with $A = 8$, $\alpha = 480/\Delta t$ and with the maximum of the intensity at $3000\Delta t$. For the simulation, eqs. (1) and (2) have been normalized in terms of the length of the nonlinear material L and the transient time τ. The parameters have then be chosen to be $\tau = n_0 L/(10c)$ as well as $\kappa L = 4$, $\beta L = 12$ and $\gamma L = 2/3$. With this choice of the parameters, the results obtained by integrating numerically along the forward and backward characteristics [2] could be reproduced. As for the discretization, for the results shown in Fig. 1, a space and time discretization interval of $\Delta z = 0.005$ and $\Delta t = 0.003125$ was chosen, thus the nonlinear material was modeled by a mesh with 200 grid points. With half of the grid points, the transient pulse starts to be distorted and is therefore not modeled correctly any more, see Fig. 2. Table 1 gives the peak values of the intensity of the transmitted pulse for modeling the nonlinear material by a mesh with 100 and 200 grid points. For the PML technique, an absorbing layer with the thickness of 100 grid points has been used.

Fig. 3 illustrates the results of the nonlinear modeling using a similar FDTD scheme. For the FDTD simulations, the same Δt as for the MRTD simulations as well as the same Gaussian excitation has been used. Furthermore, exactly the same parameters τ, κL, βL and γL as for the MRTD simulation have been chosen. The shift of the maximum of the transmitted pulse by about $500\Delta t$ is due to an additional slice of free space in the MRTD mesh between the excitation and the beginning of the nonlinear material. This additional slice of free space separates the excitation and the nonlinear material and thus allows for the use of eqs. (3) without any modifications.

As for the discretization, for the results shown in Fig. 3, a space discretization interval of $\Delta z = 0.001$ was necessary to obtain the same results as for MRTD. Thus the number of the grid points in the nonlinear material had to be enlarged from 200 for MRTD to 1000 for FDTD. Using only half of the grid points, the transient pulse starts to be distorted and thus, it is not modeled correctly any more as illustrated by Fig. 4 and by table 1. For the absorbing layer in the FDTD mesh, it was found that a thickness of 1000 discretization intervals had to be chosen in order to obtain results which were not influenced by the reflections from the absorber.

3 Conclusions

An MRTD scheme for the modeling of nonlinear pulse propagation has been derived. As for the case of linear modeling, the results suggest that the MRTD mesh can be reduced by a factor of five per dimension in comparison with FDTD while maintaining the accuracy of the field approximation. For the specific example presented in this paper, the execution time for MRTD was about a factor of 1.5 larger than for FDTD which means that the average execution time for one MRTD cell was about a factor of 7.5 larger than the average execution time for one FDTD cell. For a modeling of three-dimensional nonlinear geometries, this suggests computer savings of one order of magnitude with respect to execution time and two orders of magnitude with respect to the memory requirements. In order to exploit the full potential of MRTD, the objective of future research work is the development of MRTD schemes for the modeling of two- and three-dimensional nonlinear geometries.

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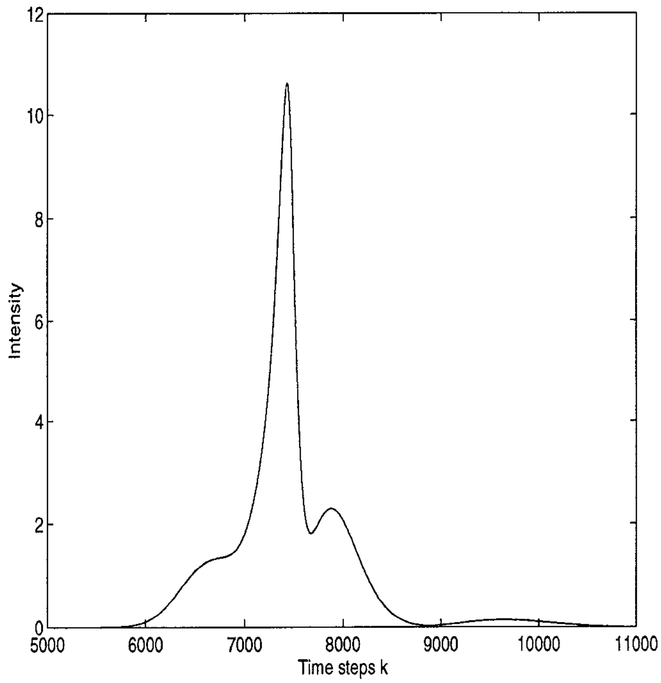

Figure 1: Transmitted pulse modeled by MRTD; 200 grid points.

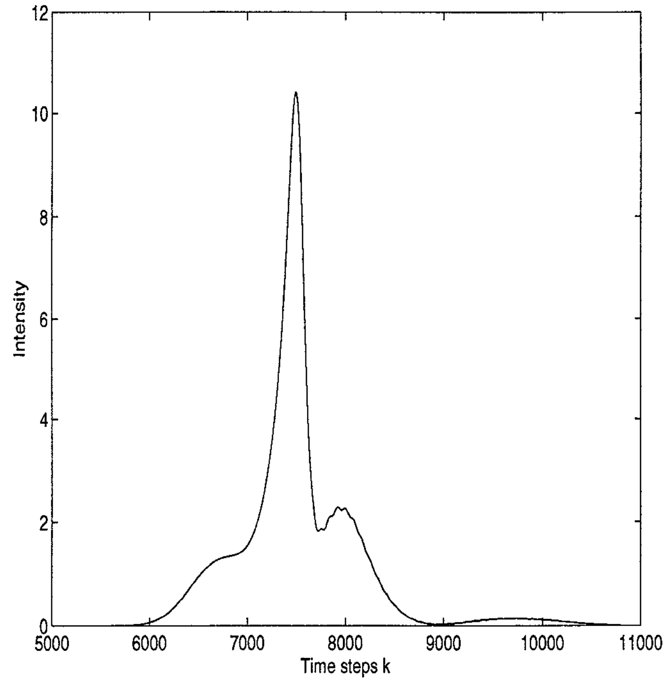

Figure 2: Transmitted pulse modeled by MRTD; 100 grid points.

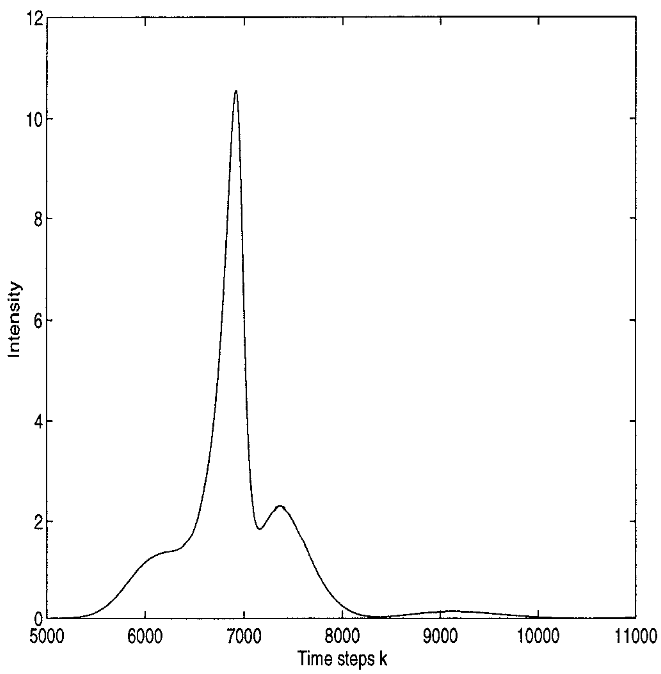

Figure 3: Transmitted pulse modeled by FDTD; 1000 grid points.

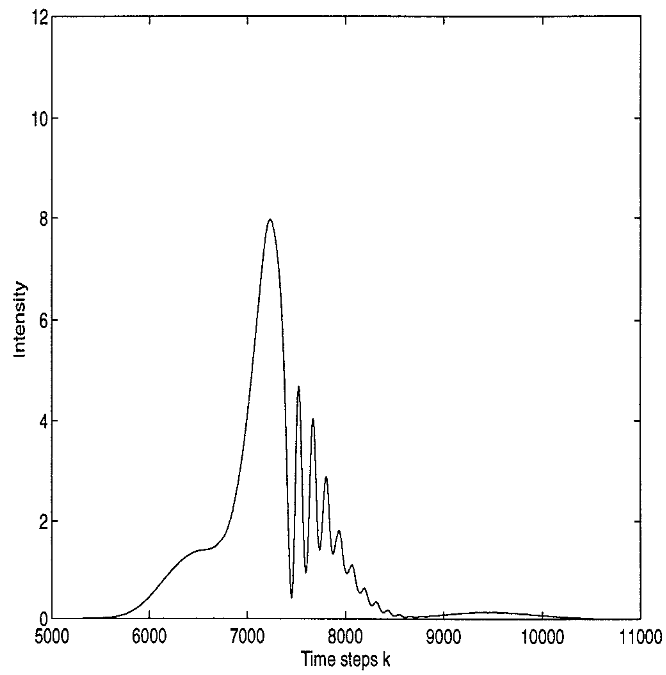

Figure 4: Transmitted pulse modeled by FDTD; 200 grid points.